

Chiral vortical effect in curved space and the Chern-Simons current

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We perform an explicit calculation of the axial current at finite temperature and rotation in curved space. We discuss a connection between the chiral vortical effect, the Chern-Simons current, and the gravitational chiral anomaly. This relation is then naturally interpreted in terms of the chiral gap effect, i.e. in terms of the shift of the fermion mass due to finite curvature. We conclude discussing possible applications to astrophysical compact objects described by the Kerr metric. We argue that the Chern-Simons chiral vortical current may provide a novel universal microscopic mechanism behind the generation of collimated jets from rotating astrophysical compact sources.

Introduction: The chiral vortical effect (CVE) refers to the well-recognized topological current induced by rotation of matter. An analytical formula for the current has been originally derived microscopically for a Dirac matter distribution in a rotating frame [1] and applied to neutrino fluxes from rotating black holes [2]. More detailed calculations for general field theories were later reported in Ref. [3]. Interest in the CVE has been recently reignited by an analogous topological phenomenon called the chiral magnetic effect (CME) that refers to the generation of an electric current due to the axial anomaly in the presence of a magnetic field (see Ref. [4] and contributions therein). From the analogy to the CME, we can naturally anticipate that the angular momentum would result in a similar effect, i.e., a chiral vortical current along the rotation axis. With the rotation axis chosen along the z direction, the chiral vortical current is written, to linear order in the angular velocity ω , as (see Ref. [5] for a recent review):

$$j_{R/L}^z = \pm \left(\frac{T^2}{12} + \frac{\mu_{R/L}^2}{4\pi^2} \right) \omega, \quad (1)$$

with R/L indicating the right-handed and left-handed sectors separately. Anomalous hydrodynamics [6], AdS/CFT correspondence [7], chiral kinetic theory [8] have all substantiated Eq. (1).

In Ref. [9] a conjecture relating the current (1) to the presence of gauge and gravitational anomalies has been proposed (see Ref. [11] for related discussions). Based on the Kubo formula for the chiral vortical conductivity with metric perturbations in the framework of fluid dynamics [12], it was shown that the coefficients of the chemical potential $\mu_{R/L}^2$ and of the temperature T^2 in Eq. (1) are respectively proportional to the chiral gauge and gravitational anomalies. It is not entirely clear whether this conjecture is true in general and it is possible to have a T^2 part of the CVE even when there is no perturbative anomaly (see Refs. [10]). However, the suggestion is intriguing as the non-vanishing transport coefficients appear as a manifestation of anomalies. It also implies that

even for an uncharged fluid the gravitational anomaly produces an anomalous CVE.

A similar connection exists between the CVE current and the Chern-Simons current j_{CS}^μ :

$$j_{CS}^\mu = \frac{1}{96\pi^2} \epsilon^{\mu\nu\rho\lambda} \Gamma^\alpha_{\nu\beta} \left(\partial_\rho \Gamma^\beta_{\alpha\lambda} + \frac{2}{3} \Gamma^\beta_{\rho\sigma} \Gamma^\sigma_{\alpha\lambda} \right). \quad (2)$$

Indeed, the formula (1) has a suggestive affinity with Eq. (2). If we perform a coordinate transformation along x and y to change ω , the current (1) should change accordingly. Such a change is possible because ω is *not* a tensor component. For example, transforming from an inertial frame to a rotating frame can induce a finite ω from $\omega = 0$, which is impossible for a tensor component. We know that the Christoffel symbols are not tensors; Γ^x_{0y} and Γ^y_{0x} have such transformation properties as expected for ω . Limiting considerations to the finite temperature part, the formula (1) should then be written more generally as $j_{R/L}^i \propto \pm \epsilon^{0ijk} (T^2/12) \Gamma^j_{0k}$. Comparing this to Eq. (2), we can observe a formal similarity, but it is puzzling how T^2 can arise. Some analyses based on holography suggest that it does [13], but a direct and field-theoretical derivation would certainly clarify and deepen our understanding of the CVE and its connection to the gravitational chiral anomaly at finite temperature.

There are two direct ways to attack this problem. One is to look at modifications of the gravitational chiral anomaly due to finite temperature effects. The other is to calculate the current expectation value explicitly in curved space at finite temperature. We have done both. Due to the slight complexity of the calculation, we will report on the former in a separate publication [14], and in the present work we will focus on the latter only, which provides a more transparent illustration of the physics. In fact, it is worth emphasizing that a curved space setup at finite T is the most natural link to connect Eq. (1) with Eq. (2), with the former involving temperature and the latter involving the geometry.

Moreover, rotating matter in curved space at finite T is not just academic fiction. Beyond the formal signifi-

cance in clarifying how thermal and geometrical effects mirror into each other, and how this impacts on anomalies, recently, it is of increasing interest to investigate the physics of relativistic rotating matter. Although the theoretical description of a spinning fluid is not yet under control [15] (see Ref. [16] for recent discussions related to the present work), microscopic field-theoretical calculations are feasible. Rotating quark matter possibly created in heavy-ion collisions may accommodate a non-trivial phase diagram as described in Refs. [17, 18]. In heavy-ion collisions, moreover, not only thermal effects but also those of strong magnetic fields play a critical role [19]. Then, as emphasized in Ref. [20], an effective chemical potential associated with rotation would *topologically* induce a non-zero density [21], which is one concrete manifestation of the chiral pumping effect [22]. (As pointed out in Ref. [23] the partition function obtained in Ref. [20] encompassed such an induced density.) Another intriguing corollary is in the context of astrophysical jets, whose origin is believed to be universal, but whose formation mechanism (i.e., acceleration and collimation) is surrounded by many open questions. It is certainly an attractive idea to draw a connection between the CVE and the microscopic nature of jets from compact astrophysical sources.

Microscopic Calculation: Our goal here is to compute the expectation value of the axial current in curved space directly using the propagator in a rotating system, i.e.

$$j_A^\mu(x) = -i \lim_{x' \rightarrow x} \text{tr}[\gamma^\mu \gamma_5 S(x, x')] . \quad (3)$$

Thus, all we need is the explicit form of the propagator $S(x, x')$ at finite T on the background of a rotating curved geometry. To construct the propagator, it is intuitively clearer to treat rotational and geometrical features separately.

We employ Riemann normal coordinates ξ around a point x (identified by $\xi = 0$) and consider the coincident limit of the fermion propagator. Using Riemann normal coordinates significantly simplifies the analysis since the Christoffel symbols at x are all vanishing and the Dirac matrices are just those in flat spacetime. A finite rotation ω can then be introduced as a small perturbation.

The coincident limit of the propagator in normal coordinates for $\omega = 0$ takes the form,

$$S_0(x, x' \rightarrow x) = \int \frac{d^4 k}{(2\pi)^4} (-\gamma^\mu k_\mu + m) \mathcal{G}(k) . \quad (4)$$

Here k is a momentum conjugate to ξ and $\mathcal{G}(k)$ is a known function involving metric derivatives [24] as

$$\begin{aligned} \mathcal{G}(k) = & \left[1 - \left(A_1 + i A_{1\alpha} \frac{\partial}{\partial k_\alpha} - A_{1\alpha\beta} \frac{\partial}{\partial k_\alpha \partial k_\beta} \right) \frac{\partial}{\partial m^2} \right. \\ & \left. + A_2 \left(\frac{\partial}{\partial m^2} \right)^2 \right] \frac{1}{k^2 - m^2} + \dots , \end{aligned} \quad (5)$$

where A_1 represents a coefficient with mass-dimension 2 that is expressed in terms of Riemann tensors at x . Analogously, $A_{1\alpha}$ is a mass-dimension 3 coefficient, and $A_{1\alpha\beta}$, A_2 are mass-dimension 4 coefficients, involving spin operators. Explicit expressions for these coefficients can be found in Ref. [24]. It is easy to argue by dimensional analysis that higher-order terms represented by the ellipses are suppressed at sufficiently high T , as we will explicitly see later.

For technical simplicity, in what follows we require two conditions to be satisfied. The first is that of stationarity (i.e. all metric components are time independent) and that the temporal components of the metric are space independent. We require this to utilize the standard Matsubara formalism valid for systems in thermal equilibrium. This condition may be relaxed at the price of using the more complicated real-time formalism to include thermal effects. Rotation induces a space-dependence in g_{00} at ω^2 order, but for our purposes it is necessary to go only to linear order in ω . This is not a particularly restrictive assumption, since, for small ω , we can always reduce the metric to a form compatible with this assumption by means of a conformal transformation.

The second condition we require is that all metric components are z independent and the z components of the metric are space independent. This condition corresponds to choosing the rotation axis along the z direction, and removes the z dependence of the spin operators of the rotation generators.

Thanks to the simplicity of Riemann normal coordinates, the calculations are straightforward. In this setup, the temporal direction is not distorted, and thus the propagator is a function of $t - t'$. This allows us to define a conjugate energy to $t - t'$ that is nothing but k_0 in Eq. (4). By applying the rotation generator, we can write the rotating propagator with ω as

$$S(\mathbf{x}, \mathbf{x}', k_0) = e^{\omega \cdot \frac{1}{2} \Sigma \frac{\partial}{\partial k_0}} S_0(\mathbf{x}, \mathbf{x}', k_0) , \quad (6)$$

for $\mathbf{x}' \sim \mathbf{x}$. (Note that \mathbf{x} is the center of rotation, so that there is no orbital term.) Here, Σ is a spin operator defined by $\Sigma^i = \epsilon^{ijk} \frac{i}{4} [\gamma_j, \gamma_k]$. Using our assumption of small ω , we can proceed to expand in powers of ω . To 0th order, we can replace $S(\mathbf{x}, \mathbf{x}', k_0)$ with $S_0(\mathbf{x}, \mathbf{x}', k_0)$. Then, using the symmetry properties of the Riemann tensors, we can readily convince ourselves that $j_A^\mu|_{\omega=0} = 0$. This is a reasonable result: even in curved space the axial current is vanishing as long as there is no rotation.

To 1st order in ω , the spin operator produces a difference leading to a non-zero Dirac trace. So, the whole quantity is proportional to $\text{tr}[\gamma_5 \gamma^\mu \gamma^{\mu'} \gamma^{\nu'} \gamma^\nu] = 4i\epsilon^{\mu\mu'\nu'\nu}$, a trait common to anomaly calculations. Some algebra gives

$$j_A^\mu = i \epsilon^{\mu\mu'\nu'\nu} \omega_{\mu'\nu'} \int \frac{d^4 k}{(2\pi)^4} \frac{\partial}{\partial k_0} k_\nu \mathcal{G}(k) , \quad (7)$$

where we used a two-index representation of the angular velocity as $\omega^i = \epsilon^{ijk}\omega_{jk}$. Using Eq. (5), we see that the first term returns the well-known formula of the CVE. That is, defining the energy dispersion $\varepsilon_k = \sqrt{\mathbf{k}^2 + m^2}$, we find,

$$\begin{aligned} \int \frac{d^4k}{(2\pi)^4} \frac{\partial}{\partial k_0} \frac{k_0}{k^2 - m^2} &= -i \int \frac{d^3k}{(2\pi)^3} n'_F(\varepsilon_k) \\ &= \frac{i}{\pi^2} \int_0^\infty dk \left(\varepsilon_k - \frac{m^2}{2\varepsilon_k} \right) n_F(\varepsilon_k), \end{aligned} \quad (8)$$

with $n_F(z)$ being the Dirac-Fermi distribution function. The integral above amounts to $\frac{i\Gamma(2)\zeta(2)}{2\pi^2}T^2 = \frac{i}{12}T^2$ in the $m \rightarrow 0$ limit, from which we correctly arrive at Eq. (1).

The most interesting correction to the axial current emerges from the second term in Eq. (5). From textbook [24] we have $A_1 = R/12$ with the momentum integration being almost the same as the previous one apart from the mass derivative:

$$\frac{\partial}{\partial m^2} \int \frac{d^4k}{(2\pi)^4} \frac{\partial}{\partial k_0} \frac{k_0}{k^2 - m^2} = -\frac{i}{2} \int \frac{d^3k}{(2\pi)^3} \frac{n''_F(\varepsilon_k)}{\varepsilon_k}. \quad (9)$$

In the $m \rightarrow 0$ limit, the above integral yields $-i/(8\pi^2)$. Therefore, together with the first term, the total current turns out to be

$$j_A^z = \left(\frac{T^2}{12} - \frac{m^2}{8\pi^2} - \frac{R}{96\pi^2} \right) \omega. \quad (10)$$

In the above, from Eq. (8), we inferred the finite- m correction to find $iT^2/12 \rightarrow i(T^2/12 - m^2/8\pi^2)$ to first order. We note that, as we discuss later, at zero temperature and zero curvature, at linear order there should be no CVE and the above expression assumes $m^2 \ll T^2$. Then, according to the chiral gap effect [25], a finite scalar curvature shifts the fermionic mass gap as $m^2 \rightarrow m^2 + R/12$, which perfectly explains the second and the third terms in Eq. (10). (See also Ref. [13] where the same term $\propto R\omega$ was discovered in a different way.)

We should point out that the coefficient $1/12$ obtained, for instance in Refs. [9, 10], is derived as the coefficient multiplying the scalar curvature term in the (heat-kernel) coefficient in Ref. [25]. Interestingly, this number, $1/12$, is universal.

We can continue the expansion to include higher-order corrections. The next contribution leading to finite corrections is $A_{1\alpha\beta}$. This term involves one more mass derivative,

$$\frac{\partial}{\partial m^2} \int \frac{d^3k}{(2\pi)^3} \left[\frac{n''_F(\varepsilon_k)}{\varepsilon_k} + \frac{n'''_F(\varepsilon_k)}{3} \right] \rightarrow -\frac{7\zeta(3)}{16\pi^4 T^2}, \quad (11)$$

in the $m \rightarrow 0$ limit. It is non-trivial that the above combination of the integrals is infrared finite, though each has singularity. This adds a correction to the current by $\delta j_A^\mu = 3\bar{A}_{100} \cdot 7\zeta(3)/(16\pi^4 T^2)$, where $\bar{A}_{1\alpha\beta}$ represents a part of $A_{1\alpha\beta}$ without spin operator [24]. In the

present treatment with only static deformations, \bar{A}_{100} is zero, however. Thus, the first non-zero correction appears from the second derivative in terms of m^2 , that is,

$$\delta j_A^z = \bar{A}_2 \cdot \frac{7\zeta(3)}{32\pi^4 T^2} \quad (12)$$

with \bar{A}_2 being a mass-dimension 4 coefficient given by $\bar{A}_2 = \frac{1}{120}R_{;\mu}{}^\mu + \frac{1}{288}R^2 - \frac{1}{180}R_{\mu\nu}R^{\mu\nu} + \frac{1}{180}R_{\mu\nu\sigma\tau}R^{\mu\nu\sigma\tau}$.

Relation to the Gravitational Chiral Anomaly: Explaining how a T -independent correction $\propto R$ in Eq. (10) arises from finite- T calculations requires a delicate interchange of the two limits; $m \rightarrow 0$ and $T \rightarrow 0$. In fact, if we keep a finite m and take the $T \rightarrow 0$ limit first, then we would have $\int \frac{d^3k}{(2\pi)^3} \varepsilon_k^{-1} n''_F(\varepsilon_k) \rightarrow 0$, and no such term survives, as we already noted. Therefore, the order of two limits, $T \rightarrow 0$ and $m \rightarrow 0$ is important. Here, we always consider the $m \rightarrow 0$ limit first and then vary T , as the value of m defines the theory, while T is a control parameter that we can adjust externally in physical situations. We will later see that the Chern-Simons current has a similar singularity at $T = 0$.

The above-mentioned calculations would be reminiscent of the high- T expansion, but we emphasize that there is a crucial difference. If one performs the high- T expansion for the pressure p for example, the leading term is proportional to T^4 , the next leading term $m^2 T^2$, and the further next term m^4 . The important point is that such m^4 term in the high- T expansion is accompanied by a logarithmic singularity, $\ln(m/\pi T)$, which blows up for both $m \rightarrow 0$ and $T \rightarrow 0$. Unlike this, in the present case, such terms involving $\ln(m/\pi T)$ exactly cancel out. The quickest interpretation for this cancellation may be presumably the fact that the anomaly receives no renormalization correction.

Let us now turn to the gravitational Chern-Simons current (2). The Chern-Simons current is not gauge invariant, but it can be argued that nevertheless it is physical. Under a coordinate transformation from x^μ to x'^μ with rotation, Γ^k_{ij} acquire a correction by $(\partial x'^k/\partial x^r)(\partial^2 x^r)/(\partial x'^i \partial x'^j)$, that gives

$$\delta \Gamma^x_{0y} = -\delta \Gamma^y_{0x} = \omega. \quad (13)$$

Then, up to linear order in ω , the Chern-Simons current takes the following form,

$$j_{\text{CS}}^\mu = \frac{\omega}{48\pi^2} (R^0_{x0x} + R^0_{y0y} - R^x_{yxy} - R^y_{xyx}). \quad (14)$$

The important observation here is that, once the ω dependence is extracted, the remaining part is written in terms of the Riemann tensors only. In our case, with flat 0 and z directions, only R^x_{yxy} and R^y_{xyx} survive, leading to our previous result, $j_A^z = -\omega R/(96\pi^2)$.

The relation between the microscopically computed current and the Chern-Simons current should be understood in the same way as for the CME current. In

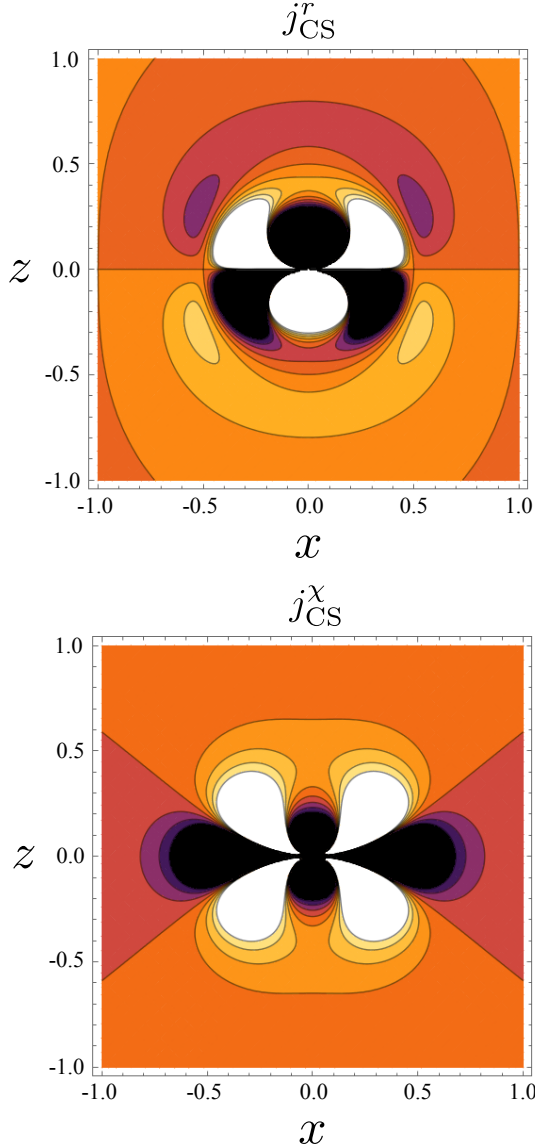


FIG. 1. Axial currents in the extremal limit. Light color represents positively large values and dark color represents negatively large values.

the CME case, the axial current along the z axis is proportional to $\epsilon^{z0ij} A_0 \partial_i A_j$, which is gauge dependent (variant). However, once the chemical potential μ is turned on, A_0 is replaced with μ , and then the rest part is the field strength tensor and thus gauge invariant; $\langle j_A^z \rangle \propto \mu B$ [26]. In this way, the Chern-Simons current can be interpreted as a physical current due to the external environment. Our explicit calculations support the idea that the argument in the derivation of the CME based on the Chern-Simons current holds also for the CVE involving the metric background with the correspondence (μ in the CME) \leftrightarrow (ω in the CVE).

Chern-Simons Current with the Kerr Metric: Once the above interpretation of the Chern-Simons current is

accepted, we have a powerful method to compute the topologically induced current. Let us consider a rotating gravitational background, described by the Kerr metric. It would be a complicated calculation to evaluate the propagator on the Kerr geometry, but it is rather straightforward to write the Chern-Simons current down. In Boyer-Lindquist coordinates $(t, r, \chi = \cos \theta, \phi)$, after some calculations, we find $j_{\text{CS}}^r \neq 0$ and $j_{\text{CS}}^\chi \neq 0$, while $j_{\text{CS}}^0 = j_{\text{CS}}^\phi = 0$. Here, instead of showing the full expressions, let us discuss j_{CS}^r and j_{CS}^χ in particular limits only. For small ω , the current to linear order in ω reads,

$$j_{\text{CS}}^r = \frac{3\pi(-3\pi + 8rT_B)\chi}{24576r^6T_B^4}\omega, \quad j_{\text{CS}}^\chi = \frac{3\pi(-1 + 3\chi^2)}{6144r^6T_B^3}\omega. \quad (15)$$

where T_B is the black hole temperature (and not the thermodynamic temperature). If the thermodynamic temperature is involved, as discussed in Ref. [13], spatial derivatives of the temperature would appear. In confronting the above expressions with the formula (1), we should remark that in Eqs. (15) both T_B and r are dimensional quantities.

The angular dependence in the above results, $j_{\text{CS}}^r \propto \chi = \cos \theta$, indicates the presence of a current aligned with the rotation axis. Coming back to the discussions in Ref. [2], we can associate this axial current with neutrino flux. It is then tempting to interpret the present results in terms of a novel (sharing some similarities with the Penrose process [27]) microscopic mechanism for the generation of collimated astrophysical jets observed in rotating compact stellar objects (see Ref. [28]). Interestingly, it may be worth noticing that this mechanism would be generic to all rotating compact objects, and not limited to black holes.

For astrophysical applications it is relevant to examine the extremal limit $T_B = 0$. One may think, using expressions (15), that the limit $T_B \rightarrow 0$ would be singular. However, before the expansion in ω , the limit smoothly exists. It should be noted that Eq. (15) gives the leading order term in an expansion in powers of ω/T_B . Therefore, we cannot extrapolate Eq. (15) naively to $T_B \rightarrow 0$. The correct result for the leading ω order in the extremal limit is

$$j_{\text{CS}}^r = -\frac{(1 - 2\xi)[\chi^4 + 4\chi^2\xi(3 - 8\xi) - 48\xi^3(1 - \xi)]\chi}{3\pi^2(\chi^2 + 4\xi^2)^5}\omega^3, \quad (16)$$

$$j_{\text{CS}}^\chi = \frac{[\chi^6 - \chi^4(3 + 56\xi^2) + 72\chi^2\xi^2(1 + 2\xi^2) - 48\xi^4]}{3\pi^2(\chi^2 + 4\xi^2)^5}\omega^4, \quad (17)$$

where $\xi = r\omega$. It is notable that these currents in the extremal limit become increasingly large for $\chi \rightarrow 0$ if ω (and ξ) is small enough. This feature is very different from Eq. (15). The currents in Eqs. (16) and (17) are

plotted in Fig. 1, where we use the unit in terms of ω and we set $y = 0$ without loss of generality due to the axial symmetry. As illustrated in the figure, the currents are strongly peaked near $z \sim 0$ (or $\theta \sim \pi/2$). It is worth noting that heavy and slowly rotating objects generally exhibits such singular structures, implying that common compact stellar objects in the universe should be accompanied by a axial currents as displayed in Fig. 1. This result implies that the CVE currents may be a source for the surrounding disk as well as the astrophysical jets.

It is an intriguing problem to discuss the physical implications of these currents for hot and dense quark matter in the heavy-ion collisions as well as for astrophysics, about which we will report in a follow-up publication [14]. In the same way as to interpret the chiral anomaly as parity-odd particle production [29], we can give a physical picture for these currents as extra contributions to phenomena similar to the Hawking radiation (see Ref. [2] for discussions along these lines). Another interesting application includes anomalous neutrino transport in rapidly rotating system of black hole or neutron star mergers (see Ref. [30] for an idea of anomalous neutrino transport in supernovae and Ref. [31] for applications to the early universe).

Summary: In this work we have calculated the axial current expectation value in curved space at finite temperature. The chiral vortical effect receives a correction proportional to the scalar curvature, R , which is consistent with the finite mass correction and the chiral gap effect. We point out that such a topologically induced current $\sim \omega R$ with ω being the angular velocity can be explained by the Chern-Simons current. Our argument parallels that in the derivation of the chiral magnetic effect that is fully explained by the replacement of A_0 with the chemical potential μ in the Chern-Simons current. This physical augmentation of the Chern-Simons current due to the external environment offers an interesting theoretical device to study the problem of particle production in non-trivial background geometries. We have applied these ideas to the case of a rotating astrophysical body and have argued that the Chern-Simons chiral vortical current may provide a novel universal microscopic mechanism behind the generation of collimated jets from rotating astrophysical compact sources.

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